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FULL WAVEFORM INVERSION USING REVERSE TIME MIGRATION AND GOLDEN RATIO

INVERSI BENTUK GELOMBANG PENUH MENGGUNAKAN REVERSE TIME MIGRATION DAN GOLDEN RATIO

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Abstract. Seismic wave imaging techniques, such as conventional full-waveform inversion (FWI), which utilize numerical solutions of seismic wave equations, can be a valuable tool for estimating high resolution models of complex geological conditions. However, conventional FWI only uses one variable recording to find the minimum misfit in the step length calculation process, unlike the FWI golden ratio, which looks for minimum misfits using the four-variable recording. The method of calculating the four-variable recording continues to be updated until the velocity value from the FWI golden ratio is close to the actual velocity model values. Then, using reverse-time migration (RTM) in this case study is crucial for determining the final results of the velocity value in the FWI golden ratio. RTM takes over as the actual recording and is used in the process of calculating the gradient on this FWI golden ratio. The gradient is then calculated with a step length to get the model update. Using the golden ratio and gradient step length from RTM really helps reduce misfit. The final result, obtained using RTM and the golden ratio in the FWI method, produces an image that resembles the shape of the true synthetic model and yields updated velocity values that are not significantly different from the true velocity values in the synthetic model.

Abstrak. *Teknik pencitraan gelombang seismik, seperti Full Waveform Inversion (FWI) konvensional, yang memanfaatkan solusi numerik persamaan gelombang seismik, dapat menjadi alat yang berharga untuk memperkirakan model resolusi tinggi dari kondisi geologi yang kompleks. Namun, FWI konvensional hanya menggunakan satu rekaman variabel untuk menemukan ketidaksesuaian minimum dalam proses perhitungan panjang langkah, tidak seperti golden rasio FWI, yang mencari ketidaksesuaian minimum menggunakan rekaman empat variabel.*

Metode perhitungan rekaman empat variabel terus diperbarui hingga nilai kecepatan dari golden rasio FWI mendekati nilai model kecepatan sebenarnya. Kemudian, penggunaan Reverse Time Migration (RTM) dalam studi kasus ini sangat penting untuk menentukan hasil akhir nilai kecepatan dalam golden rasio FWI. RTM mengambil alih sebagai rekaman aktual dan digunakan dalam proses perhitungan gradien pada golden rasio FWI ini. Gradien kemudian dihitung dengan panjang langkah untuk mendapatkan pembaruan model. Penggunaan golden rasio dan panjang langkah gradien dari RTM benar-benar membantu mengurangi ketidaksesuaian. Hasil akhir, yang diperoleh menggunakan RTM dan golden rasio dalam metode FWI, menghasilkan citra yang menyerupai bentuk model sintetis sebenarnya dan menghasilkan nilai kecepatan yang diperbarui yang tidak berbeda secara signifikan dari nilai kecepatan sebenarnya dalam model sintetis.

1. INTRODUCTION

Full-waveform Inversion (FWI) is a seismic wave imaging technique that minimizes the misfit function of the subsurface, resulting in a high-resolution model (Song et al., 2023), and uses local optimization between observed and modeled data (Jun, 2019). The FWI method simulates seismic wave propagation utilizing numerical solutions of seismic wave equations applied to complex geological conditions (Treister & Harber, 2017). Minimizing the misfit function is used to obtain the global minimum value between seismic data and subsurface model parameters (Galuzzi, 2017).

Reverse-time Migration (RTM) is a reflector migration or imaging of reflectors that uses a two-way wave equation to extrapolate the wavefield in complex geological (Zhong et al., 2019). The RTM method is the misfit gradient obtained after the first iteration and continued with FWI updating the high-wavenumber reflectivity and smooth slowness in the next iteration. The RTM method is used to migrate residual iteratively for the FWI method and gives better resolution in the image, but at the cost of being sensitive to misfits in the migration velocity model (Schuster, 2017).

Full-waveform Inversion Golden Ratio (FWI GR) is a conventional FWI method with the ratio of their sum to the larger of the two quantities. Based on the golden ratio method, the function is to find the local minimum or local maximum in an interval by using more than one point in the interval. This method can at least reduce misfits compared to conventional FWI methods (Lindfield & Penny, 2019). Meanwhile, conventional FWI methods often experience misfit spikes in several iterations due to the initial model being too far apart, resulting in cycle skipping. Conventional FWI also requires a significant number of iterations to reach its local minimum (Yang et al., 2018). FWI GR with the RTM method is expected to help reduce excessive iterations and stabilize the misfit so that it does not experience an upward spike during the search for a local minimum.

2. LITERATURE REVIEW

Ratios, specifically the golden ratio (φ) = 1,618, are found in nature (patterns of flowers, leaves, and the human body) and are applied in art, music, and architecture (Visco-Prieto et al., 2016). The golden ratio is thought to create harmony, balance, and beauty. By incorporating the golden ratio into the step lengths in FWI, it is possible to help balance misfits, thereby optimizing the approach to the global minimum. Here is its mathematical definition (Ghorbani, 2021).

$$\varphi = \frac{a+b}{a} = \frac{a}{b} \tag{1}$$

The letters (a) and (b) usually represent two lengths on a line or object that are divided in such a way that they satisfy the golden ratio. The value of φ solves the quadratic equation.

$$\varphi^2 - \varphi - 1 = 0 \quad (2)$$

So that.

$$\varphi = \frac{1+\sqrt{5}}{2} = 1{,}618 \quad (3)$$

RTM is a bidirectional seismic migration method that is more accurate than unidirectional migration methods. The basic idea of RTM is to use backpropagated and forward wavefield (shot domain) extrapolation results. RTM can handle complex geological structures by preserving amplitude and phase (Jang & Kim, 2015). RTM usually uses only the primary wave and treats multiples as noise (Lu et al., 2025). The prestack RTM image $\boldsymbol{m}(\boldsymbol{x})$ is derived from the zero-lag back-propagation correlation ($u(\boldsymbol{x}, \boldsymbol{s}, t)$) with the downgoing source field ($d(\boldsymbol{x}, \boldsymbol{s}, t)$) and produces the formula:

$$\boldsymbol{m}(\boldsymbol{x}) = \frac{u(x,s,t) \otimes d(x,s,t)}{\boldsymbol{I}(x)} \quad (4)$$

(x) is spatial location, seismic source location (s), and (t) is the time variable of the wave propagation. Illumination compensation (I(x)) with the function of stabilizing the amplitude value so that it is not too large (Schuster, 2017).

FWI is capable of producing high-resolution subsurface velocity models, but its accuracy is highly dependent on the quality of the initial models and the availability of low-frequency and long-offset data (Barnier et al., 2022). The principle of the FWI method is to minimize the residual data object function by using the difference between the actual data and the predicted data (Vigh et al., 2017). The gradient calculation is a misfit calculation obtained from the correlation between the forward wavefield (u(x,s,t)), the forward simulation trace record, and the back-propagation wavefield (d(x,s,t)) from the reverse-time process (Kim et al., 2020). The following is the equation for calculating the gradient (G) of the objective function.

$$G = s(\boldsymbol{x})^{(i)} \int_t dt \sum_s \frac{u(x,s,t) \otimes d(x,s,t)}{I(x)} \quad (5)$$

This gradient equation will then be used to determine the step length for obtaining a new model $s(\boldsymbol{x})^{(i)}$ at each iteration (Schuster, 2017; Xue et al., 2017).

3. RESEARCH METHODS

In the case study, we use two synthetic models. **Figure 1a** shows model 1 has a depth of 332 m and a distance of 812 m, with a model that resembles a layer with a simple anticline. The velocity of the first layer is 1000 m/s, the second layer is 1200 m/s, and the third layer is 1500 m/s. Then, in the second synthetic model, **Figure 1b**, with a depth of 474 m and a distance of 1194 m, the model resembles a salt diapir. This model was chosen to see that FWI GR is capable of quite complex layers. Four layers that have velocity in each layer: 1000 m/s, 1200 m/s, 1400 m/s, and 1500 m/s. The choice of velocity only represents the difference in an artificial lithology. Synthetic model 1 and model 2 utilize 13 sources and 51 receivers. **Figure 2** shows a stacked RTM image synthetic model with a single-shot RTM setup, Acoustic Finite-Difference Time Domain (AFD TD).

AFD TD uses finite-difference second order, with each time step value in each synthetic model being 0.001 seconds. The grid spacing of synthetic model 1 is 5m, and that of synthetic model 2 is 12m. The absorbing method uses a sponge boundary method that functions to dampen waves at the edge of the domain, so that they do not bounce back and produce artificial reflections.

Calculate the gradient (G) From Equation (5) by using reflection from RTM as a back-propagation wavefield. and calculate with the step length golden ratio. The calculation is as follows.

First, define the initial step length (a_0) as $a_0 = a_a = 0$, and define the step length integers (a_n) as $a_n = a_b = n$ (integers). Calculate another step length c (a_c) and d (a_d), with the golden ratio (φ),

$$a_c = a_a + \varphi^2 (a_b - a_a) \quad (6)$$

$$a_d = a_a + \varphi (a_b - a_a) \quad (7)$$

Update Slowness used each step length ($s(x_a), s(x_b), s(x_c), s(x_d)$), and ($s(x_0)$) is the initial slowness,

$$s(x_a) = s(x_0) + a_a G \quad (8)$$

$$s(x_b) = s(x_0) + a_b G \quad (9)$$

$$s(x_c) = s(x_0) + a_c G \quad (10)$$

$$s(x_d) = s(x_0) + a_d G \quad (11)$$

Calculate the misfit (f_a, f_b, f_c, f_d) from each Slowness Update ($s(x_a), s(x_b), s(x_c), s(x_d)$) and observed data ($d_{obs}$),

$$f_a = s(x_a) + d_{obs} \quad (12)$$

$$f_b = s(x_b) + d_{obs} \quad (13)$$

$$f_c = s(x_c) + d_{obs} \quad (14)$$

$$f_d = s(x_d) + d_{obs} \quad (15)$$

If misfit $f_c \leq f_d$, interval $[a_a, a_d]$, $a_b = a_d$,

$$a_d = a_a + (1 - r)(a_b - a_a) \quad (16)$$

If misfit $f_c \geq f_d$, interval $[a_c, a_b]$, $a_a = a_c$,

$$a_c = a_a + (1 - \varphi)(a_b - a_a) \quad (17)$$

End if misfit $s(x)^{(i)} - a_a G <$ misfit $s(x)^{(i)} - a_b G$.

This equation indicates that the model update aligns with the true model, and the model update converges to the global minimum (i.e., the true model) (Ghorbani, 2021; Schuster, 2017; Xue et al., 2017).

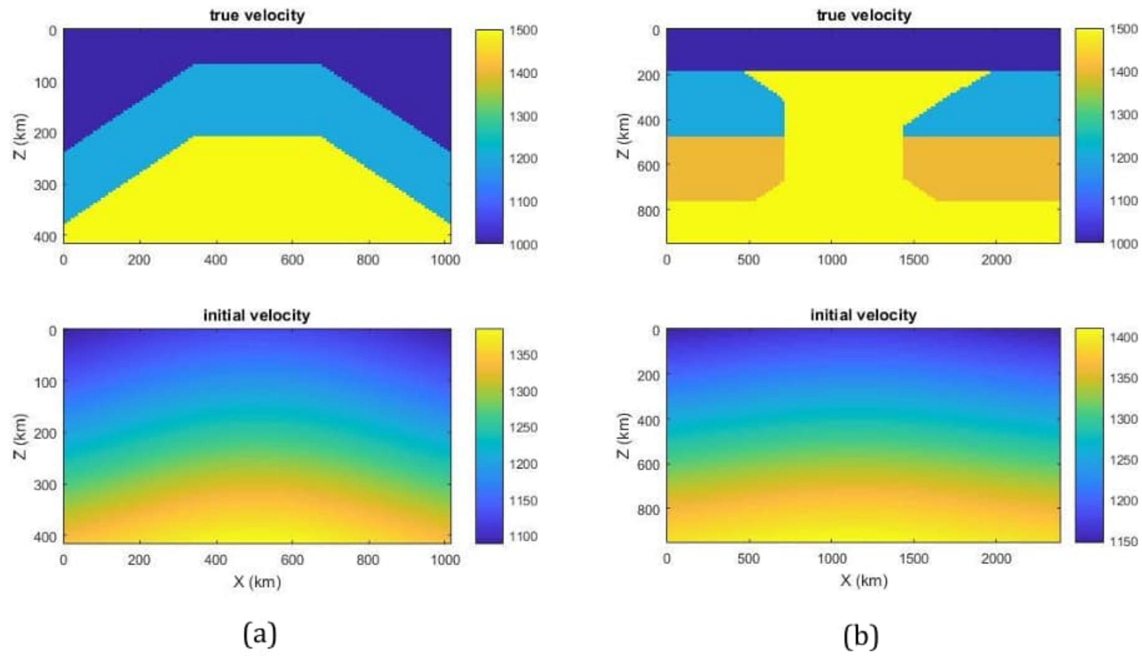

Figure 1. True velocity and initial velocity (a) synthetic model 1, (b) synthetic model 2.

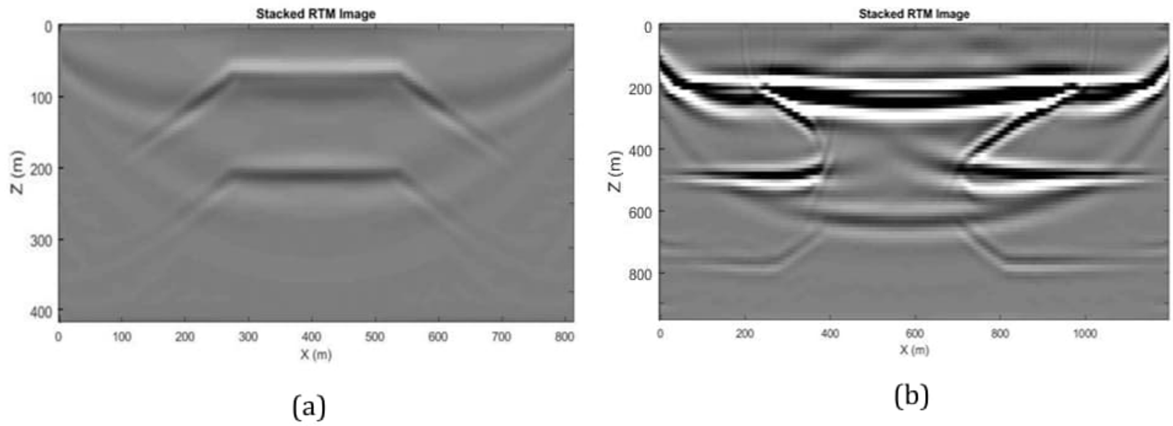

Figure 2. Stacked RTM image (a) synthetic model 1 with three layers, (b) synthetic model 2 with four layers.

4. RESULT AND DISCUSSION

Misfit calculation obtained from a run forward simulation on the background model that produces trace records. The trace record is then reduced by the true record, which is obtained from the RTM process. Calculation of misfit update when solving the wave equation using the test model in the step length calculation process using the golden ratio, which results in an updated misfit. The updated misfit obtained is normalized to the misfit in the initial model (initial misfit price), as shown in **Figure 3**. The effect of large and small misfits depends on how well the RTM is generated.

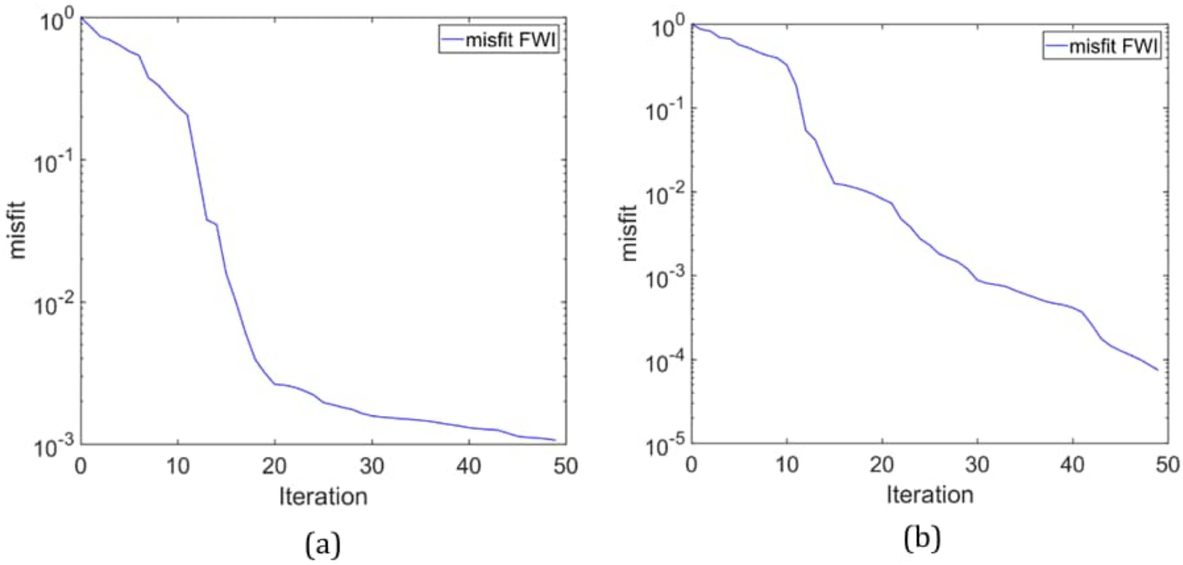

Figure 3. Misfit from (a) synthetic model 1, (b) synthetic model 2.

Figure 4 and **Figure 5** show the velocity profiles at two different locations using two different models. For synthetic model 1, the horizontal distances are 270 m and 520 m. For synthetic model 2, the horizontal lines are 520 m and 1740 m. The two velocity profiles in synthetic model 1 show that the FWI GR is almost the same as its true velocity, but at depths of 300–400m, the difference is not very significant. It is because a single ricker wavelet (10Hz) cannot cover the bottom depth. If we use a higger frequency from the first iteration, it introduces a multiple in the final image (FWI model at 50 iterations) (Aparajita & Winardhi, 2022). The effect of significant velocity differences in the depth of 0 – 270 m in the velocity profile of synthetic model 1 is influenced by how well the RTM is generated, as well as when determining the misfit. Velocity profile at 1740 m, the velocity profile shows that the FWI GR is almost identical to the true velocity. We know it is because the synthetic model 2 is deeper than the synthetic model 1, and also the layer from the synthetic model 2 is wider than the layer of the synthetic model 1.

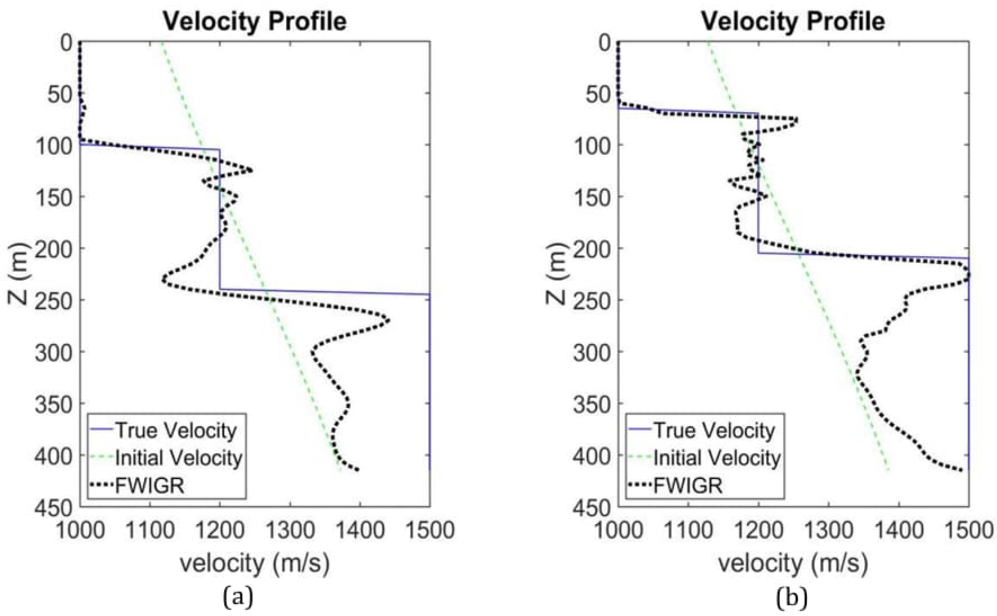

Figure 4. Velocity profile comparison synthetic model 1 at distance (a) 270 m, (b) 520 m.

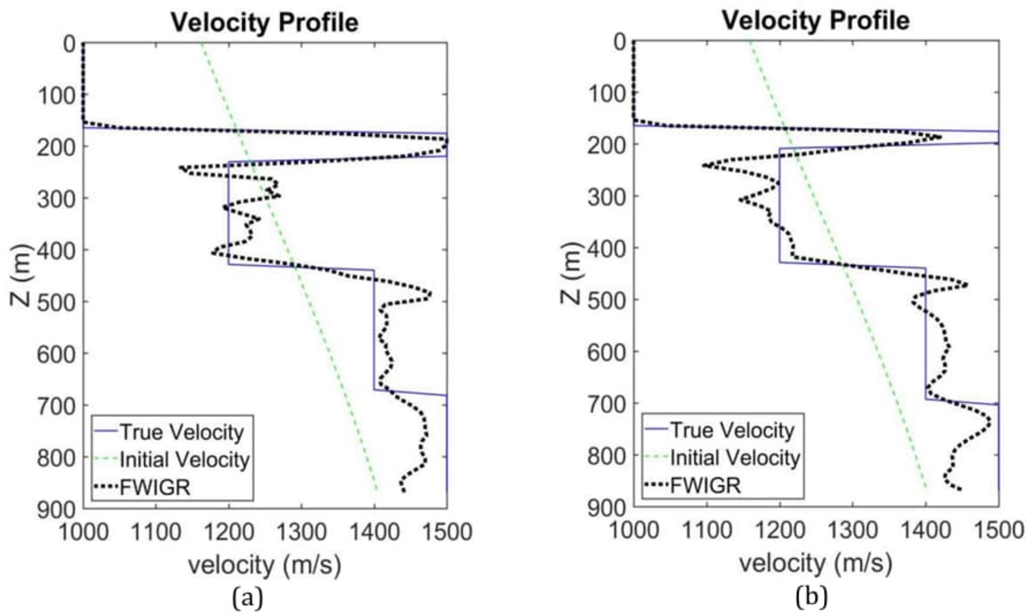

Figure 5. Velocity profile comparison synthetic model 2 at distance (a) 520 m, (b) 1740 m.

So that fewer waves are reflected, and minimizes the multiple effects (because it uses a single frequency at the Ricker wavelet). **Figure 7** and **Figure 9** show the final results of the velocity update from the initial model in **Figure 1** using the FWI obtained from the RTM and the Golden ratio. In synthetic model 1, FWI GR from **Figure 6** and **Figure 7**, with 50 iterations and only a single frequency, in iterations 1-10, we can see that the resulting image is still far from its original form or true velocity. This is due to the initial velocity being designed far from the true velocity. However, the resulting misfit in **Figure 3** experienced a very significant decrease and did not experience an increase in misfit. This indicates that the step length used can reduce cycle skipping so that it still provides an appropriate path from processing (close to the true velocity). Iterations 11-20 also continued to provide significant help, but in iterations 21-50, there was a decrease in misfit that was not substantial.

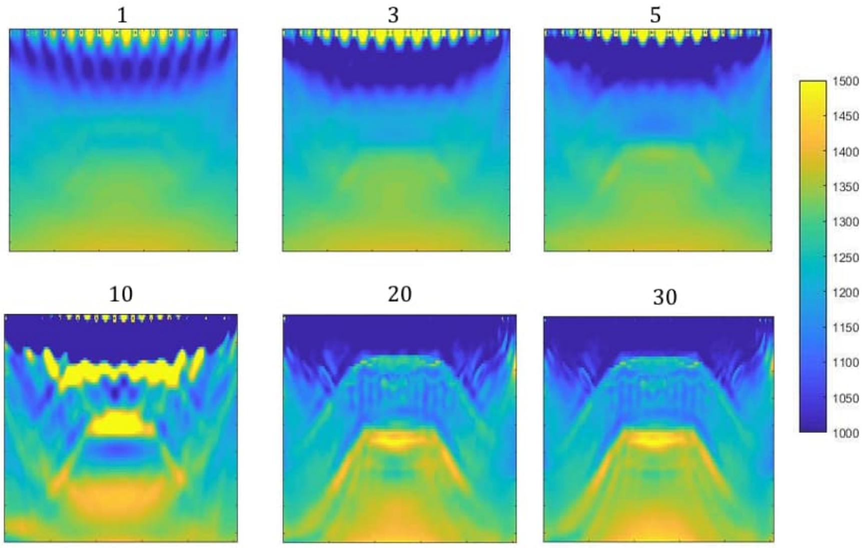

Figure 6. Updated velocity full-waveform inversion golden ratio iteration 1, 3, 5, 10, 20, 30 synthetic model 1.

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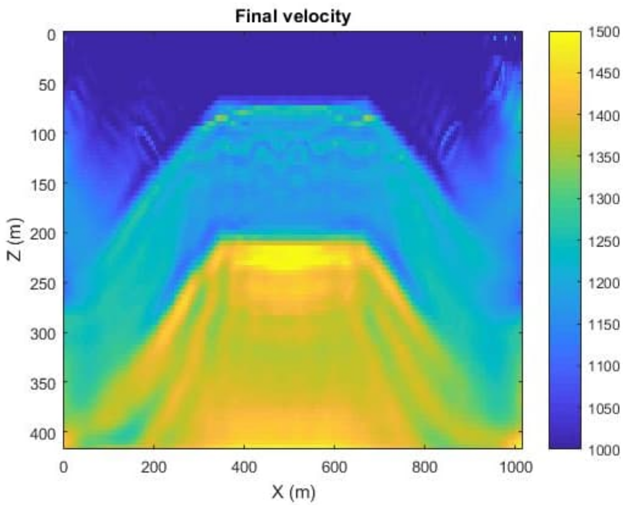

Figure 7. Updated velocity full-waveform inversion golden ratio iteration 50 synthetic model 1.

This is because the accuracy level of the golden ratio step length has reached its optimal solution point. Similarly, in **Figure 8** and **Figure 9**, a significant decrease was only in iterations 10-15. This also still affects synthetic model 2, which has deeper and more complex layers than synthetic model 1, as seen in **Figure 1a** and **Figure 1b**.

From this research, FWI GR with RTM can help reduce misfit, preventing sudden increases in misfit in subsequent iterations (Pladys et al., 2021) and preventing it from stopping at the wrong local minimum. The iterations required to reach the correct local minimum (close to the initial model) are also not as many as previous research papers (Pladys et al., 2021) and (Yang et al., 2018). These limitations and sensitivities are first in AFD TD, where acoustic waves still ignore S-waves, anisotropy, and P-S wave conversion. Then, small numerical reflections sometimes appear even though absorbing boundary conditions have been used, which can affect the results of the model inversion. Of course, the FWI method is sensitive to the initial model. If FWI uses the initial model too far, it increases the chance of obtaining a wrong local minimum (cycle skipping).

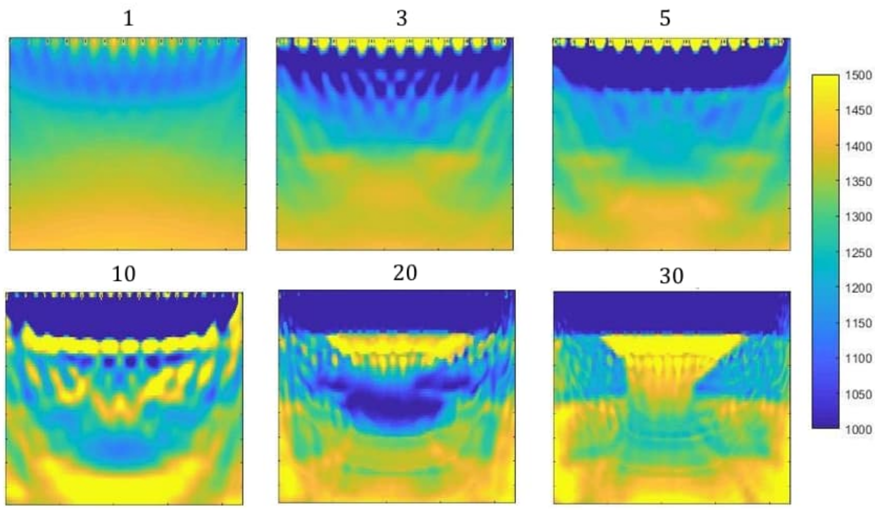

Figure 8. Updated velocity full-waveform inversion golden ratio iteration 1, 3, 5, 10, 20, 30 synthetic model 2.

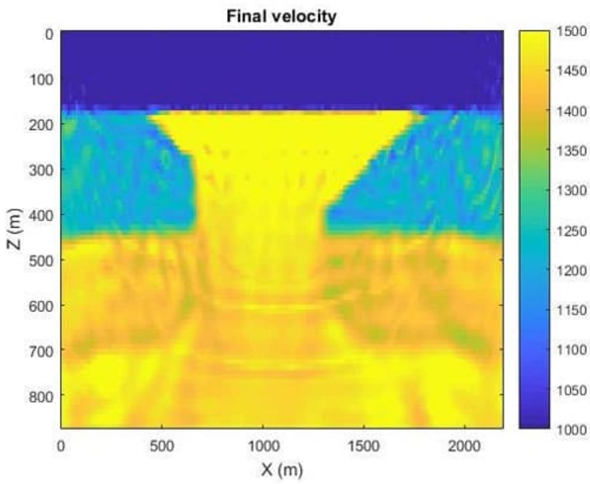

Figure 9. Updated velocity full-waveform inversion golden ratio iteration 50 synthetic model 2.

However, this research at least helps avoid cycle skipping and the final velocity profile FWIGR produced from synthetic model 1, **Figure 4** and synthetic model 2, **Figure 5** is still close to the true velocity. Moreover, the final velocity of these two synthetic models still reflects the shape of the true velocity. This research, in the future, can help with real data issues by solving problems with cycle skipping and reducing misfit, which results in stopping at the wrong local minimum.

5. CONCLUSION

This case study concluded that the importance of determining RTM and the golden ratio in the FWI GR, RTM method, as a gradient provides the direction of the descent so that the update will be effective. Using the golden ratio for step length significantly helps reduce misfit consistently and avoids an increase in misfit values. The golden ratio concept and gradient from the RTM method create balance and consistency. Although the final velocity results are quite good, several additional methods are needed to achieve a more optimal final velocity imaging.

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